Development and laboratory validation of in-line multiplexed low-coherence interferometric sensors

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Abstract

In this paper, we present the development and laboratory validation of in-line multiplexing for a low-coherence interferometric strain sensor suitable for industrial deployment and application to civil structures. The sensor is the multiplexed version of the standard SOFO, developed, produced and commercialized by Smartec SA. While the standard SOFO employs total reflectors at the end of the measurement and reference fibers, allowing measurement of the strain only over a single field, in the solution presented in-line multiplexing is obtained separating each measurement field by partial reflectors, consisting of broadband Fiber Bragg Gratings (FBGs) with a 5% reflectivity. Laboratory tests have been carried out on a prototype 3-field sensor, to investigate effectiveness, resolution and temperature sensitivity. Outcomes show a linear response of the sensor with RMS resolution lower than 3 µm, independent of the measurement base, of the same order as the single field sensor. Consistently with the theoretical prediction, the system exhibits an apparent thermal expansion coefficient of 2 µε°C−1, relatively low if compared with the thermal expansion coefficient of steel or concrete structures. This temperature dependency can even be eliminated by appropriate selection of the length of the reference fiber. Theoretical analysis indicates that the maximum number of fields that can be arranged in series is in the order of 10; however this limit can be overcome by appropriately selecting the power of the light source of the interrogation unit.

Keywords: Long gauge-length strain sensor; SOFO; In-line multiplexing; Fiber Bragg grating; Temperature sensitivity

1. Introduction

Structural health monitoring of large civil structures, such as long-span bridges or multi-story buildings, often involves the use of a large number of sensors, in some cases in the hundreds: in such circumstances, connecting sensors to the reading unit can be a very demanding task. Moreover, the high installation and maintenance costs of traditional electrical sensors and their reduced durability tend to limit their application in long term monitoring of civil structures. One of the advantages of fiber optic over traditional gauges is that they can act both as sensors and as a pathway for signals produced by other sensors: this lets us use a simple fiber optic sensor (FOS) system architecture such as the in-line multiplexing scheme, even when arrays of many sensors are needed. Good sources documenting the progress in this field are, for instance, Udd [1], Ansari [2,3], Leung [4], Measures [5], Mufti [6], Li et al. [7], Lee [8].

Among optical fiber sensor technologies, narrowband fiber Bragg gratings (FBGs) have attracted considerable attention since they provide relatively high strain resolution and multiplexing capability, and are virtually insensitive to fluctuations in source power [1,5,9]. For the narrowband FBG sensors, a multiplexing scheme is generally achievable by two techniques: wavelength division multiplexing (WDM) [1,5,10] or time division multiplexing (TDM) (see, for example, [11,12]). The former needs gratings tuned at different wavelengths and a wide transmission band, while the latter implies launch and detection of short wave-packets. An issue in large scale deployment of FBG-based sensor systems is cost: signal recovery requires complicated demodulation techniques, usually involving expensive hardware. Another major drawback to the insertion of narrowband FBG in concrete structures is the small size of the gauges. As their size typically ranges from a few millimeters to

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a few centimeters, FBGs are perfectly suited to measuring local strain, but inappropriate for elongation measurements in concrete structures, where measuring over a longer base (from 0.5 to a few meters) is typically required.

In order to overcome this problem, one option is to mount the short gauge on an intermediate structure which in turn is coupled to the structure to be monitored at only two points. There are many examples of these attempts in the literature [5]. The first two authors themselves recently developed and tested a long gauge-length sensor for direct displacement measurements based on FBG [13]. This sensor model basically consists of a 600 mm protected acrylate-coated fiber including a grating. It looks like a flexible wire and can be easily handled and fitted to the monitored structure by bolting its heads to metal supports. As another example, Blue Road Research and the University of California, San Diego, have been collaborating over several years to develop a system employing FBG strain sensors and modal analysis to provide real-time quantitative information on dynamic bridge response [14]. Other FBG-based FOS (either long or short gauges) are suggested by Maalej et al. [15]. Whatever the solution, it is clear that the effectiveness of the system is based on the assumption that the intermediate structure will deform uniformly, so that the strain measured at the FBG can be extended to the whole length of the gauge.

Alternatively to this indirect measuring method, a number of long gauge-length sensors have been also proposed, many of them based on the low coherence interferometric principle. Interferometer sensors do not need an intermediate structure for integration into concrete, and there is no need to prestress the measuring fiber, insofar as the measurement fiber can be directly glued to the structure or to a rigid support. The advantage of using low coherence instead of high coherence interferometry is that absolute measurements are possible, not only incremental, because interference occurs only within the coherence length. In addition, they allow accuracy in the order of the µm, independent on the measurement base.

Among all the systems proposed in the literature, particularly promising for future deployment seems to be the work by Yuan et al. [16–18]. If we restrict our attention to interferometric sensors that are commercially available and suitable for integration into civil structures, the SOFO (Surveillance d’Ouvrages par Fibres Optiques) system, developed and commercialized by Smartec SA [19], is possibly the best known representative of this family. As better explained in the next section, SOFO is based on low-coherence double Michelson interferometers in tandem configuration. To date the SOFO system has been applied to bridges, dams, tunnels and in the laboratory, to monitor both the local properties of the construction materials and the overall behavior of the structure [20–22].

A completely different family of long gauge-length sensors is based on Brillouin scattering effect [5]. This principle is currently implemented in the form of two types of sensing processes, referred to as the Brillouin optical time-domain reflectometry (BOTDR) and Brillouin optical time-domain analysis (BOTDA). Both systems today allow us to determine the strain profile along a single mode fiber up to 10 km long, with a typical spatial resolution of about 1 m and with a maximum estimated theoretical resolution of the order of 10 cm for BOTDA, according to Brown et al. [23]. Compared to other FOS technologies, the main drawback of Brillouin sensors is their limited strain accuracy, which is of the order of 30 µε for BOTDR [24] and 10 µε for BOTDA [23] with current interrogation technologies. These features still make BOTDR/BOTDA systems suitable for inexpensive distributed monitoring of large scale structures, at least in those cases where strain accuracy and exact damage location are not critical issues: the scientific literature reports successful applications to piles, pipes, tunnels, levees, dams, steel structures, and steel cables [24–27].

The long-term durability of optical fiber sensors in a concrete environment is another issue of major concern, as their performance is strongly affected by the fiber/matrix interface. Leung et al. [28] investigated systematically the interfacial changes of fibers with different coatings embedded in mortar specimens under various environmental conditions and proposed a pull-out test technique for the characterization of the time dependent interfacial behavior. The integration of FOSs in construction materials is often limited by the difficult handling of bare fibers: to overcome this problem, Glisic and Inaudi [29, 30] presented a novel packaging system consisting of a composite tape which can be easily manipulated, embedded and surface mounted. Fernando et al. [31] investigated several sensor protection systems, both numerically and by experiment, including stainless steel, concrete, glass- and carbon-fiber packaging. Following the same project, Kister et al. [32] embedded protected FOS in RC columns at different positions and investigated their response to failure. Focusing on aerospace applications, Betz et al. [33] studied different methods for attaching FBG short-gauge FOS to the host structure: bonding the fiber to the surface of the structure, embedding in fiber-reinforced composites and integrating into paint. Based on numerical analysis and experimental tests, they supply information on the influence of bond type and distribution on sensor response. In general, we observe that integration is easier and durability is enhanced when the packaging does not require the fiber to be in a permanent state of tension. As mentioned, for long base measurements this condition can be easily fulfilled using interferometric sensors.

Despite this merit, to date commercial interferometric sensors do not feature in-line multiplexing, although this possibility was investigated theoretically by Inaudi et al. [34] as from 1998. In this paper, we present the development and laboratory validation of in-line multiplexing for a SOFO standard deformation sensor. This work is part of a research effort aimed at new bridge construction using smart structural elements, defined as precast reinforced concrete elements embedding a sensing system and capable of self-diagnosis [35]. Sensors are conceived as an integral part of the prefabricated element, influencing its design criteria and detailing.

In Section 2, we provide a brief explanation of the working principle of the standard SOFO with special attention to the issue of temperature compensation. Then in Section 3 we explain how the same measurement concept can be extended to an in-line multiplexing scheme, and we propose a technological solution where broadband FBGs are employed as partial reflectors. To investigate the effectiveness, resolution and stability of this so-
In this section we summarize the measurement principle exploited by the standard SOFO. All these concepts are extensively explained in Inaudi’s Ph.D. thesis [22], to which the reader is referred for details. The basic concept of the SOFO sensor is that of a Michelson interferometer. In this scheme a light signal is split up into two optical paths ending with reflectors. Different path lengths result in a different phase lag of the returning beams which, once recombined, will interfere in a constructive or destructive manner. The return signal intensity then provides information on the difference in the optical path lengths.

The SOFO measurement system is based on the principle of path-matching interferometers, as illustrated in Fig. 1. The sensor itself is a Michelson interferometer, below referred to as a sensor interferometer. One of the two interferometer arms is loose, while the other is pre-tensioned and connected to the monitored structure in such a way as to follow its same strain variation. A broadband (i.e., low-coherence) light signal is injected into the sensor and the signal from the sensor is processed by a second, analyzing interferometer. This latter device has one arm fixed and the other ending with a movable mirror controlled by a micrometer. In this way the phase lag can be modified by moving the mirror. The measurement is performed by scanning the light intensity at all possible positions of the mirror.

The interrogation unit makes use of a low-coherence Gaussian broadband source, with a central wavelength \( \lambda_0 = 1310 \text{ nm} \) and a bandwidth \( \Delta \lambda = 40 \text{ nm} \). The corresponding coherence length is \( l_c = \frac{\lambda_0^2}{\Delta \lambda} = 43 \text{ \mu m} \). In terms of propagation constant \( k \), the spectrum \( s_0(k) \) of the source can be described by a central value \( k_0 = \frac{2 \pi}{\lambda_0} = 4.8 \text{ \mu m}^{-1} \) and a bandwidth \( \Delta k = 2 \pi / l_c = 0.15 \text{ \mu m}^{-1} \), as to the following expression:

\[
 s_0(k) = \frac{1}{\sqrt{2 \pi} \Delta k} \exp \left\{ -\frac{1}{2} \left( \frac{k - k_0}{\Delta k} \right)^2 \right\}. \tag{1}
\]

Hereinafter, it is convenient to express the dimension of an arm in terms of equivalent optical path \( d \), defined as the path covered by light in a vacuum in the same time taken by a signal to travel forward and backward in the arm. Evidently, for a fiber optic arm, calling \( L \) the physical length of the arm and \( n \) the index of refraction of the fiber core, we have

\[
 d = 2 n L. \tag{2}
\]

The difference in equivalent length in the sensor and the analyzing interferometers can be expressed as

\[
 \Delta d_{AB} = d_A - d_B, \tag{3a}
\]

\[
 \Delta d_{CD} = d_C - d_D, \tag{3b}
\]

where indices A and B refer to the measurement arm and reference arm of the sensor interferometer, respectively, while C and D refer to the fixed and movable arms of the analyzing interferometer. As illustrated in Fig. 1, during the measuring process the source signal \( s_0(k) \) is first modulated by the sensor interferometer, then by the analyzing interferometer. The intensity of the signal received by the detector is given by the sum of a constant term and of a term depending on the position of the movable mirror \( \Delta d_{CD} \). The corresponding fringe visibility \( V \) has the following expression:

\[
 V \propto 2 \exp \left\{ -\frac{1}{2} (\Delta k \Delta d_{CD})^2 \right\} + \exp \left\{ -\frac{1}{2} (\Delta k (\Delta d_{CD} + \Delta d_{AB}))^2 \right\} + \exp \left\{ -\frac{1}{2} (\Delta k (\Delta d_{CD} - \Delta d_{AB}))^2 \right\}. \tag{4}
\]

The fringe visibility, seen as a function of the scanning position \( \Delta d_{CD} \), exhibits three peaks: one central for \( \Delta d_{CD} = 0 \), and
two lateral, of half amplitude, for $\Delta d_{CD} = \pm \Delta d_{AB}$. The former occurs when the two arms of the analyzing interferometer have equal optical path length, while the latter appears when the delay $\Delta d_{CD}$ counterbalances the delay introduced by the sensor interferometer $\Delta d_{AB}$. In conclusion, to perform a measurement, it is sufficient to find the position of the three intensity peaks in the mirror scan: the value of $\Delta d_{AB}$, depending on the structural strain $\varepsilon$, is related to the physical displacement $p$ between the mirror positions generating the central and the lateral peaks.

2.2. Strain and temperature sensitivity of the measurement

Above we analyzed the sensor mechanism in terms of equivalent optical path delay $d$. Now we need to analyze how the quantity $\Delta d_{AB}$ is measured by the physical mirror shift $p$, on one side, and how it is influenced by the actual sensor strain $\varepsilon$ on the other. The first relation is quite straightforward. Because the environmental conditions are maintained constant in the interrogation unit, the mirror physical shift $p$ is univocally linearly related to the delay $\Delta d_{AB}$ through a calibration constant:

$$p = \frac{1}{2n_{CD}} |\Delta d_{AB}|,$$  

where constant $n_{CD}$ can be seen as an equivalent index of refraction for the analyzing interferometer. For instance, $n_{CD} = 2.0$ for the Smartec interrogation unit used in the experiment reported in Section 4. Note that in Eq. (5), $\Delta d_{AB}$ is expressed as an absolute value, because the system is unable to distinguish the sign of the path unbalance.

The relation between the optical delay $\Delta d_{AB}$ and sensor strain needs deeper discussion, first because both strain $\varepsilon$ and temperature $T$ affect the refractive index $n$ of the sensor fibers, then because temperature change alters the physical length of the reference fiber even in the absence of strain. It is therefore important to theoretically quantify how changes in strain $d\varepsilon$ and temperature $dT$ at the sensor affect the system measurement $p$. Considering only the linear effects, an incremental variation $dp$ of the measurement can be formally expressed as

$$dp = \left[ \frac{\partial p}{\partial \varepsilon} \right]_T d\varepsilon + \left[ \frac{\partial p}{\partial T} \right]_\sigma dT,$$

where indices $T$ and $\sigma$ under the partial derivatives indicate that these values are calculated at the reference temperature or stress, respectively. Let us first analyze the effect of a pure strain variation $d\varepsilon$. Preliminarily, it is convenient to rewrite Eq. (5) as

$$p = \frac{1}{2n_{CD}} |\Delta d_{AB}| = \frac{|n_AL_A - n_BL_B|}{n_{CD}}.$$  

In order to remove the sign ambiguity of the term under absolute value, let us initially suppose the path unbalance $\Delta d_{AB} > 0$; which is to say the case where the measuring fiber is longer than the reference one, assuming $n_A$ roughly equal to $n_B$. A strain variation is ineffective on the reference fiber, but changes both the physical length $L_A$ and the refractive index $n_A$ of the measuring fiber, due to the strain-optic effect. Neglecting the second-order terms, we easily end up with the following relation for the partial derivative of $p$ respect to $\varepsilon$:

$$\left[ \frac{\partial p}{\partial \varepsilon} \right]_T = \frac{1}{n_{CD}} \left\{ n_A + \left[ \frac{\partial n}{\partial \varepsilon} \right]_T \right\} L_A = \frac{1}{n_{CD}} S_{\varepsilon} L_A$$

where evidently constant $S_{\varepsilon}$ has been here defined as

$$S_{\varepsilon} = \frac{1}{L_A} \frac{d(n_AL_A)}{d\varepsilon} = n_A + \left[ \frac{\partial n}{\partial \varepsilon} \right]_T.$$

Using a similar approach we can calculate the effect of a pure temperature variation. For the pre-tensioned fiber $A$ we should consider both the thermo-optic effect and the strain-optic effect on $n_A$ caused by fiber relaxation. As to the reference fiber $B$, we should account for the thermo-optic effect on $n_B$ and the thermal expansion of the fiber, using a thermal expansion coefficient $\alpha_{Fc}$ for the coated fiber, which is typically higher than that of the glass core $\alpha_{F}$. In addition, following a thermal expansion, the different thermal expansion coefficients of coating and core will produce an elastic strain $d\varepsilon = (\alpha_{Fc} - \alpha_{F}) dT$ on the fiber. Accounting for all these effects, the temperature dependency of the sensor can be expressed in the form:

$$\left[ \frac{\partial p}{\partial T} \right]_\sigma = \frac{1}{n_{CD}} \left\{ \left[ \frac{\partial n}{\partial T} \right]_\sigma - \left[ \frac{\partial n}{\partial \varepsilon} \right]_F \alpha_{F} \right\} L_A$$

$$- \left( n_B \alpha_{Fc} + \left[ \frac{\partial n}{\partial T} \right]_\sigma \right) + \left[ \frac{\partial n}{\partial \varepsilon} \right]_F (\alpha_{Fc} - \alpha_{F}) \right\} L_B$$

$$= \frac{1}{n_{CD}} (S_{T,A} - \delta S_{T,B}) L_A$$

having defined the rate:

$$\delta = \frac{L_B}{L_A}$$

and the temperature sensitivities:

$$S_{T,A} = \frac{1}{L_A} \frac{d(n_AL_A)}{dT} = \left[ \frac{\partial n}{\partial T} \right]_\sigma - \left[ \frac{\partial n}{\partial \varepsilon} \right]_F \alpha_{F}, \quad (12a)$$

$$S_{T,B} = \frac{1}{L_B} \frac{d(n.BL_B)}{dT} = n_B \alpha_{Fc} + \left[ \frac{\partial n}{\partial T} \right]_\sigma + \left[ \frac{\partial n}{\partial \varepsilon} \right]_F (\alpha_{Fc} - \alpha_{F}). \quad (12b)$$

In conclusion the strain and temperature dependency of measurement $p$ can be summarized as

$$dp = \left[ \frac{\partial p}{\partial \varepsilon} \right]_T d\varepsilon + \left[ \frac{\partial p}{\partial T} \right]_\sigma dT$$

$$= \frac{1}{n_{CD}} \left\{ S_{\varepsilon} d\varepsilon + (S_{T,A} - \delta S_{T,B}) dT \right\} L_A.$$  

It is convenient to express temperature influence using an apparent thermal expansion coefficient of the sensor $\alpha_s$, defined as

$$\alpha_s = \frac{S_{T,A} - \delta S_{T,B}}{S_{\varepsilon}}$$

so that relation (13) can be formally rewritten as

$$dp = \frac{1}{n_{CD}} S_{\varepsilon} (d\varepsilon + \alpha_s dT) L_A.$$  

At this point we must go back to Eq. (7) and remember that this result has been derived assuming $\Delta d_{AB} > 0$. If, on the contrary,
Δd_{AB} < 0, we simply observe that in this case all incremental changes produce opposite effects on the path unbalance, and therefore on p.

The last two relations are useful to estimate the actual temperature disturbance to the mechanical measurement of the sensor. Representative values for the opto-thermo-mechanical quantities appearing in Eqs. (9) and (12) are given in the literature. Measures [5] suggests use of α_T = 0.5 με °C⁻¹ as the thermal expansion coefficient for bare glass fibers, n_A = n_B = 1.46 as refractive indexes and [dn/δε]_T = −0.32 and [dn/δT]_T = 10 με °C⁻¹ as partial derivatives respect to strain and temperature, respectively. Using these values, we obtain sensitivities of S_T = 1.15, S_{T,A} = 10.16 με °C⁻¹. Coefficient α_{Fc} depends on the coating type and thickness and on the diameter of the fiber. For instance, rough estimates of the expansion coefficient are α_{Fc} = 0.9 με °C⁻¹ for a single mode 145 μm polyimide-coated fiber and α_{Fc} = 3.2 με °C⁻¹ for a single mode 250 μm acrylate-coated fiber. With these assumptions, the corresponding values for the thermal sensitivity are S_{T,B} = 11.19 με °C⁻¹ for the polyimide fiber and S_{T,B} = 13.80 με °C⁻¹ for the acrylate fiber. Note that the thermal sensitivity of the reference fiber S_{T,B} is generally different from that of the measuring fiber S_{T,A}, implying the temperature effect is not perfectly compensated even when the two fibers have identical length (δ = 1). In this case, what we theoretically expect is an apparent thermal expansion coefficient α_a = −0.89 με °C⁻¹ for the polyimide fiber and α_a = −3.17 με °C⁻¹ for the acrylate fiber, where the minus sign indicates that the sensor apparently contracts when the temperature increase. These are only roughly indicative values: in reality it is difficult to predict accurately the apparent thermal coefficient using Eq. (14), because some of the quantities involved, and notably α_{Fc}, are very uncertain. Conversely, it is more convenient to directly calibrate α_a through experimental tests, as later illustrated in Section 4.4. In any case, we can observe that the expected values of α_a are generally tolerable for civil application, where the thermal expansion coefficient of the host structure is in the order of 11 με °C⁻¹ for steel, 12 με °C⁻¹ for concrete, and 23 με °C⁻¹ for aluminum. They are also much lower than that observable on uncompensated narrowband FBG, estimated in the order of 10 με °C⁻¹ using the same assumptions as before.

Equation (14) suggests that in theory perfect thermal compensation is obtained by selecting an appropriate dimension for the reference fiber, so that the rate δ is equal to the optimum value:

\[ \delta_{opt} = \frac{S_{T,A}}{S_{T,B}}. \]

With the sample values assumed before, we obtain δ_{opt} = 0.91 for a 145 μm polyimide-coated fiber and δ_{opt} = 0.74 for a 250 μm acrylate-coated fiber, which is to say that to eliminate the temperature disturbance the reference fibers should be 9 and 26% shorter than the measuring ones, respectively. Again, we need to stress that these values are just rough estimates of the optimal rate δ, to be taken as indicative values.

3. Multiplexed SOFO

3.1. Concept

A limit of the interferometric deformation sensor as presented in the previous section is that the strain is measured over a single field. As already suggested by Inaudi [21,22], the general idea to extend this principle to a multiplexed scheme is to divide the two optical paths of the sensor interferometer into a number of measurement fields by partial reflectivity mirrors (PRMs). A possible scheme of this in-line multiplexed arrangement is shown in Fig. 2: in this case, pairs of PRMs delimit the first N = 1 measurement fields, while the line is closed by a pair of total reflectivity mirrors at the end of this chain.

The measuring concept resembles the principle of the single field interferometric sensor discussed in Section 2.1. As schematically illustrated in Fig. 3, each pair of corresponding mirrors defines a reference and a measurement path. Suppose that the path unbalance for the kth pair of mirrors is ΔD_{AB,k}: according to Eq. (4), the fringe visibility of the signal received by the analyzer will include peaks for Δd_{CD} = ±ΔD_{AB,k}. The same reasoning applies to each of the PRM pair, thus in conclusion the fringe visibility is the superposition of the responses of the N single interferometers.

In detail, note that the position p_k of the kth visible peak is sensitive to any change occurring at the fields preceding the kth pair and is given by the following expression:

\[ p_k = \frac{\sum_{j=1}^{k} (n_{A,j} L_{A,j} - n_{B,j} L_{B,j})}{2 n_{CD}} \]

\[ = \frac{\sum_{j=1}^{k} \Delta d_{AB,j}}{2 n_{CD}}, \]  

where L_{A,j} and L_{B,j} are the physical lengths of the jth field on the measuring and reference arms, n_{A,j} and n_{B,j} are the

Fig. 2. In-line multiplexed SOFO sensor.
corresponding index of refraction and \( \Delta d_{AB,j} \) denotes the optical path unbalance produced by the \( j \)th field. Before proceeding, note that we can remove the absolute value assuming \( \Delta D_{AB,k} > 0 \) for each \( k \)th pair. Note that this condition is automatically assessed when \( \Delta d_{AB,j} > 0 \) for every field: with indices of refraction approximately identical, this is to say we have the measuring arm longer than the reference, on every field. In a similar manner we see that if the condition \( \Delta D_{AB,k} < 0 \) is fulfilled for every \( k \), then we can remove the absolute value in Eq. (17) after changing the sign: such a situation is verified, for instance, when the references are all longer than the measuring fields. If one of the two aforementioned conditions is fulfilled, then, the difference \( f_k \) between the peak positions inherent to two consecutive mirror pairs reduces to the following simple relation:

\[
f_k = p_k - p_{k-1} = \pm \frac{n_{A,j}L_{A,j} - n_{B,j}L_{B,j}}{nCD},
\]

where the plus/minus sign reminds us that an incremental change of strain may result in a positive or negative \( dp_k \), depending on the sign of the optical path unbalance. Following the same logical path in Section 2.2, we can easily derive the relation between incremental changes in strain \( d\varepsilon_j \) or temperature \( dT_j \) on any \( j \)th field and the variation of the peak distance \( df_k \) associated to the \( k \)th and \((k-1)\)th mirror pairs. Specifically,

\[
df_k = dp_k - dp_{k-1} = \pm \frac{1}{nCD} S_e (d\varepsilon_k + \alpha_{A,k}dT_k) L_{A,k},
\]

where \( \alpha_{A,k} \) is the apparent thermal expansion coefficient for field \( j \). Note that this equation is formally similar to the corresponding expression valid for the single SOFO provided by Eq. (15). A very important point highlighted by Eq. (19) is that

the field measurement \( f_k \) depends only on the properties and the state of the \( k \)th field, while it is insensitive to action applied to fields other than \( k \). It is worth mentioning that the apparent thermal expansion coefficient depends on the rate \( \delta_j = L_{B,j}/L_{A,j} \), and therefore generally may change with the field even when the refractive index can be assumed uniform along the array. Here, as in the single SOFO, we can select theoretically appropriate dimensions for the reference line in such a way as to eliminate the thermal effect. However, the length of the reference fiber also determines the baseline value of peak separation \( f \), and in practice this has to be chosen in order to keep the measurement within the physical dimension of the mirror range. This issue will be better discussed in Section 3.3.

3.2. Technological solution

A first technological problem for implementing the proposed concept is how to obtain a partial reflectivity mirror within a fiber. Inaudi [21, 22] suggested various solutions for reproducing this effect: for instance, some types of defect cause partial reflections, as well as changes in the reflective index along the fiber. However, the main issue is how to control by design the reflectivity, and how to limit the intensity loss that these solutions involve. In the prototype presented in this paper, we decided to use broadband fiber Bragg gratings (FBGs). FBGs generally work as low-loss PRMs on a selected wavelength window. In essence, their reflective behavior is characterized by three parameters: the central reflective wavelength \( \lambda_c \), the reflective bandwidth \( \Delta \lambda \) and the reflectivity \( R \). These parameters in turn depend on the grating length, the grating pitch, the chirp rate and the modulation amplitude of the reflective index. The target FBG for our application should act approximately as a wavelength blind PRM in the whole range of the light source (\( \lambda_0 = 1310 \text{ nm}, \Delta \lambda = 40 \text{ nm} \)): hence the selected target parameters of the grating are \( \lambda_c = 1310 \text{ nm} \) and \( \Delta \lambda = 50 \text{ nm} \). Also, a target reflectivity \( R = 5–7\% \) was specified, for reasons that will be better explained in the next section. To fulfill these specifications, a chirped FBG was designed, using a phase mask of 908 nm central pitch, and a chirp rate of 8.75 nm/cm, using the \( \pm 1 \) order principle [5]. Fig. 4 reports the reflective spectrum experimentally obtained from one of these gratings, while its reflectivity was measured as \( R = 6.2\% \). The first measurement tests demonstrate a clear fringe visibility for multiple pairs of gratings: for example, Fig. 5 shows the experimental fringe visibility of a 3-field setup, similar to that later introduced in Section 4. We also observe that the peak width \( w \) is of the order of 30 \( \mu \text{m} \), roughly double that obtained with a classic single SOFO sensor, operating with total reflectivity mirrors.

3.3. Limits of the technology

An issue with an in-line multiplex scheme, deserving special attention, is the maximum number of fields that can be arranged in series. This number is generally limited by two facts. First of all, while in the single-field sensor the light injected is totally reflected by two mirrors, in the multiplex scheme it is partially reflected by several pairs: the most unfavorable case is that of
Fig. 4. Reflected power spectrum from a broadband FBG used in the sensor prototype (from AOS GmbH).

Fig. 5. Experimental fringe visibility obtained on a 3-field prototype sensor.

the signal returning from the last PRM, which has to pass all the other mirrors twice. When the number of reflectors is high, the intensity of the returning beam might be too low to be detected by the receiver. A second problem is related to the position of the interference peaks: to reconstruct the strain profile along the fields, we must avoid peak overlap and at the same time all peaks must fall within the physical measurement range.

To deal with the first problem, assume a series of identical PRMs, each with reflectivity $R$; also consider an overall insertion loss $L$ for each reflector and therefore a transmissibility $T = (1 - R - L)$. With such hypotheses, the intensity of the back signal $r_k$ returning from the $k$th mirror in the row is given by the following expression:

$$r_k = RT^{2(k-1)}.$$  

(20)

Evidently, in order to minimize the loss, it is more rational to end the optical lines with two total reflectivity mirrors (TRMs), instead of partial reflectivity ones, as suggested in Fig. 3. Being $N$ the total number of fields, in this case the intensity of the back signal $t_N$ from the last TRM is

$$t_N = T^{2(N-1)}.$$  

(21)

Although this arrangement optimizes the back intensity from the most critical mirror, yet it breaks the modularity of the system, and for this reason the solution adopting PRMs at all field is, in the opinion of the authors, the most promising in view of industrial deployment of the sensor. Fig. 6 shows the values of $r_k$ and $t_N$ for different values of reflectivity $R$, assuming a loss $L = 0.2$ dB. Of course, the optimum value for $R$, i.e., the value that provides the highest back signal along the whole series, depends on the number of reflectors. For example if only 3 reflectors are employed, it is convenient to adopt $R = 20\%$; however, for a series of 10 reflectors it is more convenient to adopt $R = 5\%$. The same figure highlights that the TRM at the end of the chain is typically more visible than the last PRM. More specifically, from Eqs. (20) and (21) we can deduce that this is always true up to $R = 37\%$. The maximum number of fields depends on the sensitivity of the interrogation unit compared with the light source: for example, with the source and detector currently mounted in the SOFO interrogation unit, a loss of 20 dB seems to be tolerable [22]. For 10 measurement fields, arranging nine partial plus one total reflectivity mirror for each fiber, and using $R = 5\%$ we obtain $r_9 = 1.05\%$, corresponding to a loss of 19.8 dB respect to the source. Evidently, the limit of 10 reflectors can be overcome by increasing the power of the light source or the sensitivity of the detector. In any case, note that for real life installations we have to account also for the signal attenuation in the optical line from the source to the sensor, including all the extrinsic losses due to couplers, connectors and switches, if any. These additional losses can be negligible or not, as to the complexity of the optical setup and the scale of the monitored structure. Also environmental conditions and the design lifespan of the monitoring system can be critical when the excess losses due to ageing cannot be neglected. Thus, the maximum number of fields calculated in this paragraph has to be taken as an indicative value, while in general the correct dimensioning of the optical components of the sensing system is an issue that has to be carefully evaluated case by case.

The problem of peak overlap can be tackled having in mind the scheme represented in Fig. 7. First note that in general the peak sequence detected by the movable mirror does not reflect the corresponding sequence of fields. Indeed, the actual peak position basically depends on the cumulative optical path unbalance. Although there is no special drawback in not having the following requirement fulfilled, for the sake of clarity in the discussion that follows we will suppose that in each field the
peak separation \( f_k \), the condition to avoid a peak overlap is to have the baseline conservation of the PRM pairs and an elongation of range on the \( j \)th field increase the value of \( f_k \). We will also assume that \( \alpha_{u,k} > 0 \) for every \( k \). Let us say that the expected measurement range on the \( j \)th field is \((-\Delta\varepsilon_j^{(-)}, \Delta\varepsilon_j^{(+)})\), while the expected temperature excursion is \((-\Delta T_j^{(-)}, \Delta T_j^{(+)}\).

A first problem is that when the two peaks are not sufficiently separated, peak \( k - 1 \) might overlap peak \( k \). Evidently, the condition to avoid a peak overlap is to have the baseline peak separation \( f_{k,0} \) greater than the maximum expected contraction of field:

\[
f_{k,0} \geq \Delta f_k^{(-)} + w = \frac{1}{n_{\text{CD}}} S_\varepsilon (\Delta\varepsilon_k^{(-)} + \alpha_{u,k} \Delta T_k^{(-)}) L_{A,k} + w,
\]

where \( w \) is the minimum peak resolution that can be conservatively taken as equal to the peak width. Observe that the quantity \( w \), of the order of few \( \mu \)m, is normally negligible respect to \( \Delta f_k^{(-)} \), which typically ranges from few millimeters to few centimeters.

A further problem is that when all the sensor fields experience the maximum elongation, then the farthest peak might fall outside the effective measurement range \( u \), being \( u \) the distance between the central visibility peak position and the closer stroke-end of the movable mirror. The general condition to prevent this effect is

\[
u \geq \frac{1}{n_{\text{CD}}} \sum_{j=1}^{N} (\alpha_{u,j} \Delta T_j^{(+)}) L_{A,j} + w.
\]

Even selecting appropriate values of \( f_{k,0} \), we obtain at least:

\[
u \geq \frac{1}{n_{\text{CD}}} \sum_{j=1}^{N} (\alpha_{u,j} \Delta T_j^{(+)}) L_{A,j} + (N + 1)w,
\]

where \( \Delta\varepsilon_j = \Delta\varepsilon_j^{(-)} + \Delta\varepsilon_j^{(+)\text{ is the total expected strain measurement range; and similarly for } \Delta T_j. \text{ When the strain effect is dominant respect to the thermal deviation and peak resolution, Eq. (24) brutally simplifies to}

\[
u \geq \frac{S_\varepsilon}{n_{\text{CD}}} \sum_{j=1}^{N} (\Delta\varepsilon_j L_{A,j}) L_{A,j} = \frac{S_\varepsilon}{n_{\text{CD}}} \Delta\varepsilon L_A,
\]

where \( \Delta\varepsilon \) is the average strain range over the total measuring length \( L_A \) of the multiplexed sensor. Note that the minimum value of \( u \) basically depends on the total length of the sensor and the expected strain, but is independent of the number of fields. For example, assuming \( S_\varepsilon = 1.15 \) and \( n_{\text{CD}} = 2 \), Eq. (25) tells us that if we want to interrogate a 10 m long sensor and the expected strain range is a uniform \( \Delta\varepsilon = 1\% \), then the mirror stroke should be at least 58 mm, regardless of the actual number of fields.

All these results have been derived assuming all measuring paths to be longer than the references, and positive thermal expansion coefficients. If, on the contrary, all the reference fibers are longer than the measuring ones, we must exchange \( \Delta\varepsilon_j^{(-)} \) with \( \Delta\varepsilon_j^{(+)\text{ and } \Delta T_j^{(-)} \text{ with } \Delta T_j^{(+)\text in the same equations. In any case, following the same logical path, we eventually reach equations identical to (24) and (25).
4. Experimental validation

4.1. Test setup

To investigate the effectiveness, resolution and stability of the proposed in-line multiplex solution, a sensor prototype was developed and tested in the laboratory. In essence, the sensor consists of a measuring and a reference 145 µm polyimide-coated fiber, each including three broadband FBGs with characteristics specified in the previous section. As shown in Figs. 8 and 9, the test setup comprises three 1 m-long measurement fields: the elongation of each field can be controlled, independently of the others, by a linear stage with resolution of 1 µm. Within a measurement field, both measurement and reference fibers are arranged in a single plastic protection tube, adopting a packaging similar to that of a SOFO standard deformation sensor. In this case the measuring fiber is pretensioned between two points, although this is not strictly required. The three pairs of broadband FBGs are in plastic boxes, placed between the measurement fields. The same commercial reading unit available for single SOFO sensors was used to launch, receive and analyze the optical signal in the sensor, while the intensity response function was digitally recorded using a laptop. The mechanical setup allows imposition of a specific elongation, and therefore a specific strain rate, to each of the three fields. Using this setup, a number of tests was carried out to investigate effectiveness, precision and temperature dependency of the multiplex SOFO scheme: the main results are reported in the following sections.

4.2. Sensor response

A first set of tests was carried out to verify linearity and precision of the sensor strain response. For instance, Fig. 10 shows the results of a test where stages are moved one at a time, in such a way as to impose strain variations on the single fields, while keeping the other two fields unchanged. Consistent with Eqs. (15) and (19), in this and in the following graphs the measured peak shifts \( \Delta p_k^* \) and field elongations \( \Delta f_k^* \) are those calculated taking into account constant \( n_{CD} \) and strain sensitivity \( S_\varepsilon \), which is to say:

\[
\Delta p_k^* = \frac{n_{CD}}{S_\varepsilon} (p_k - p_{k,0}),
\]

\[
\Delta f_k^* = \frac{n_{CD}}{S_\varepsilon} (f_k - f_{k,0}),
\]

where \( p_{k,0} \) and \( f_{k,0} \) are the baseline peak position and the baseline field, respectively. We can see immediately that when the measurement fiber is appropriately pre-tensioned (in this specific case for elongation greater than 4 mm for the first field) the elongation measured on each field is very close to that actually imposed by the mechanical stage. As another example, Fig. 11 shows the results of a test in which all three fields are simultaneously changed. Evidently in this case, the corresponding elongations are mutually correlated, because the elongation of each field is calculated as the difference between two peak positions: however, precision and linearity of the measurements do not deteriorate.

4.3. System precision

The system precision was evaluated by keeping the linear stages fixed and making a hundred consecutive measurements. It is worth noting that the measurements, even if consecutive, are not correlated, because the interrogation unit generates a new visibility plot at each mirror scan. Fig. 12 shows the cumulative histogram of the measurement noise, having defined...
Fig. 10. Experimental results of a strain test, deforming a single field at a time.

Fig. 11. Experimental results of a strain test, deforming all fields at the same time.

Fig. 12. Distribution of measurement noise for one hundred repeated measurements.

noise as the difference between system measurement and actual mechanical elongation imposed at the micrometric stage. Table 1 shows the statistical parameters of the measurements. According to these tests, the RMS precision of the sensor is always lower or equal to 3 µm, while the strain precision, obtained dividing this value by the measurement base $l = 1.0$ m, is in this case lower or equal to 3 µε. It is interesting to observe that this value is somewhat higher than the nominal precision of the standard single SOFO, which is 1.5 µm. It is also worthy of note that this precision is expected to be independent of the measurement base.

Another interesting result of the statistical analysis of noise is that noise levels associated with two different peaks are somewhat correlated. This is evident from the calculation of
Table 1  
Statistical outcomes of the precision analysis test

<table>
<thead>
<tr>
<th>Peak position</th>
<th>Average (mm)</th>
<th>RMS (µm)</th>
<th>Variance (µm^2)</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak 1</td>
<td>18.654</td>
<td>2.511</td>
<td>6.305</td>
<td>1.346 x 10^{-4}</td>
</tr>
<tr>
<td>Peak 2</td>
<td>6.133</td>
<td>1.916</td>
<td>3.670</td>
<td>3.124 x 10^{-4}</td>
</tr>
<tr>
<td>Peak 3</td>
<td>20.544</td>
<td>2.705</td>
<td>7.315</td>
<td>1.317 x 10^{-4}</td>
</tr>
<tr>
<td>Field 1</td>
<td>18.654</td>
<td>2.511</td>
<td>6.305</td>
<td>1.346 x 10^{-4}</td>
</tr>
<tr>
<td>Field 2</td>
<td>12.521</td>
<td>3.005</td>
<td>9.031</td>
<td>2.400 x 10^{-4}</td>
</tr>
<tr>
<td>Field 3</td>
<td>14.411</td>
<td>2.578</td>
<td>6.646</td>
<td>1.789 x 10^{-4}</td>
</tr>
</tbody>
</table>

The statistical correlation \( \rho_{ij} \) between the \( i \)th and the \( j \)th peak position, defined as

\[
\gamma_{ij} = \frac{\sigma_{ij}^2}{\sigma_i \sigma_j},
\]

where \( \sigma_i \) is the standard deviation of the \( i \)th peak, while \( \sigma_{ij}^2 \) indicates the covariance associated to the \( i \)th and \( j \)th peaks. The values calculated for each pair of peaks are \( \rho_{12} = 0.11 \), \( \rho_{23} = 0.16 \), and \( \rho_{13} = 0.48 \), respectively. These values are always positive, meaning that deviations associated with a peak pair are more likely to have the same sign. This experimental observation is consistent with expectation: in fact, we must remember that all three peak positions are estimated with respect to the same central peak in the fringe visibility, thus a deviation in the estimate of the central peak affects identically all the peak measurements. The same observation also explains why the variance of the elongation measurement on a field is lower than the sum of the variances of the corresponding peaks.

4.4. Temperature sensitivity

The thermal response of the sensor was investigated by a specific laboratory experiment: in the test the first sensing field was heated from 26 to 35 °C, while maintaining the elongation in each field unchanged. Special attention was paid to confining the heated area to the sensor only, in order to avoid unwanted thermal expansion of the rigid support of the test setup. For the same reason, the test time was limited to 20 min. Fig. 13 reports the temperature and sensor response history as well the resulting correlation between temperature and measured peak position, and the experimental relation between peak position and temperature. In the temperature/peak position graph, the slope of the fitting line, i.e., the thermal sensitivity of the sensor, is expected to be equal to the product \( \alpha_A L_A \), where \( \alpha_A \) is the apparent thermal coefficient introduced in Section 2.2 and \( L_A = 1.00 \text{ m} \) is the measurement base of the field heated during the test.

As shown in Fig. 13, all peaks respond experimentally to temperature with a sensitivity of approximately \( -2 \mu \text{m}\text{°C}^{-1} \), corresponding to an apparent thermal expansion coefficient \( \alpha_a = -2 \mu \text{m}\text{°C}^{-1} \). The fact that all peaks show an almost identical negative slope is in line with the theoretical discussion of Section 3.1. Note that a temperature change on the first field causes almost identical shift \( \Delta f_k \) to all three peak positions, simply because this is the first field in the optical array. Evidently, if we reason in terms of field measurement \( \Delta f_k \), we observe the temperature effect on the first field, but not on the others. This is fully consistent with Eq. (19), whereby the field measurement \( \Delta f_k^* \) depends only on the state of the \( k \)th field, while it is insensitive to any temperature or strain change on fields other than \( k \).

5. Conclusions

A novel in-line multiplex sensor based on Michelson interferometer technology has been developed and experimentally validated. This sensor adopts broadband fiber Bragg gratings as partial reflectivity mirrors, and can be interrogated by the same standard low-coherence reading unit currently offered by Smartec for interrogating single SOFO sensors. The theoretical analysis suggests that the maximum number of fields that can be arranged in series in a single line is of the order of 10, the

![Fig. 13. Outcomes of the thermal test: relations between time, temperature, and peak position.](image-url)
limitation being the intensity of the returning light signal. Evidently, this restriction can be overcome by increasing the power of the light source or the sensitivity of the light detector. In the prototype developed, we used identical FBGs as partial reflectivity mirrors, thus the reflection peaks can be recognized based on their position on the fringe visibility graph. This fact limits the strain range available to each measurement field. In theory, to overcome this difficulty, one might adopt reflectors having differing characteristics, each producing a specific peak profile; however, the practical feasibility of this idea has yet to be demonstrated.

Laboratory experiments have been carried out to investigate the system performance in terms of linearity, resolution and thermal stability. The outcomes show that the RMS resolution of the system is lower than 3 μm. This means that the multiplex SOFO is slightly less accurate than the standard single SOFO, where the nominal precision is 1.5 μm. As in the standard SOFO, the precision is independent of the measurement base. As to temperature sensitivity, tests show an apparent disturbance effect is small and negligible if compared with the thermal expansion coefficient of steel or concrete structures, and it is suggested that it can even be eliminated by appropriate selection of the length of the reference fiber.

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References


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